The calculation of rope-rod structures of ropeways on the basis of the new approach

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Abstract: *In this paper the method of simulation and calculation of complex rope – rod structures, basing on the discrete representation of rope and rods in the form of the set of point masses, springs and dampers is given. For its realization the algorithm is developed. On the basis of corresponding geometry of a body, the values of efforts, displacements of units and other physical parameters are determined.*

Key words: *rope; rod; simulation; calculation.*

INTRODUCTION

The object of consideration of this presentation, in the first place, is a rope – rod structure, which can be defined as the set of pulleys, rollers, shoes, hinges, supports and other standard lumped structural elements, interacting and connected with ropes and rods and forming an integral three-dimensional deformable structure.

The approach of simulation and calculation of solid deformable bodies we examine for the so-called rarefied rope – rod structures (RRS), a characteristic feature of which is a comparatively small mass with large geometric dimensions and deformations.

A good example of rarefied RRS is an ropeway, without which it is difficult to present the infrastructure of contemporary tourism. The three-dimensional constructions, which contain as a basic element ropes, rods, and/or their combinations with lumped structural elements, besides cableways, are encountered in various spheres of engineering practice. These are transport and hoists, building structures, sea economy, electro technical constructions, military science, space technology. They are realized also in the form of cranes and elevators, floating boring platforms, suspension bridges, passages, conduits and overlaps, orbital cable systems [1, 2].

Rarefied RRS, as a rule, are stretched for many hundreds of meters, in certain cases, even for tens of kilometers, for example, the so-called dumbbell systems in the space, which have complex configuration, and are subjected to nonstationary dynamic effects and large deformations.

These constructions are characterized by high costs and complexity. Therefore questions of simulation and optimal planning (calculation) of such constructions are urgent scientific problems, whose results are important not only for ropeways but in many branches of engineering practice.

During the simulation of complex and, in particular, rarefied RRS it is necessary to consider the special features characterizing them: large values of deformations, which fall outside of the limits of linear properties of materials, many degrees of freedom, complex forms of actions by loads and disturbances, the set of steady states, presence of nonlinearity, friction, hysteresis. The problems of mathematical description (simulation) are aggravated by the presence of substantially nonlinear connections, for example, transverse sags, elastic-plastic deformations, irregular nature of a change in the coefficient of friction, thermal deformations, breakage of characteristic functions, nonlinearly changing mobile boundary conditions (for example, displacement of a cabin near the support of ropeway).

Classical approach in the solution of simulation (calculation) problem of similar complex three-dimensional systems consists in the composition of mathematical description (correct description, generally speaking, is possible on a basis of means of differential equations in partial derivatives) and of numerical solution of the corresponding system of equations.

The problems of simulation of similar systems have mathematical nature (weak convergence with "bad" initial approximations), or they are connected with excessively long machine time. Frequently both these problems appear simultaneously and are equally significant. The way out of situation is in the linearization of connections and in the decrease of a quantity of degrees of freedom. This means moving to the less precise model and, therefore, obtaining less accurate results and, in the final analysis, reducing real safety factors and an increase in the risk of emergency. The aforesaid is confirmed by the cases, when the carriers of similar objects, after putting into commission, with large loads and deformations were additionally strengthened or even generally broken.

DISCRETE MODEL OF ROPE AND ROD

Below it is described the method of computer simulation and calculation of complex RRS, developed by the author of this article at the Institute of Mining Mechanics of Georgian Academy of Science [3, 4, 5] during creation of applied programs (CAD) and investigation of ropeways and orbital space cable systems.

Although this approach has a claim on universality, further we will examine this approach for the example of classical cableways and suspension roads and conduits, which for complexity and variety of forming structures can be, perhaps, considered as ideal object from the point of view of checking applicability and propagation of the proposed method for the simulation and calculation of analogous objects from other areas of technology.

In a basis of the approach is put *the discrete model of rope and rod*. Fundamental components of rope - rod structures: arbitrarily loaded rope and rod are represented exclusively as the set of movably or rigidly connected discrete standard components, which are point masses, elastic elements (springs) and dampers.

In Fig. 1 a is shown the rope sagged in the gravitational field (P_g), arbitrarily loaded, for example, by distributed wind (R_w) and one concentrated (P_c) load, while in Fig. 1 b – the corresponding discrete model (in order to keep clearness, the dampers of elementary sections, intended for the dissipative properties of rope, are not represented).

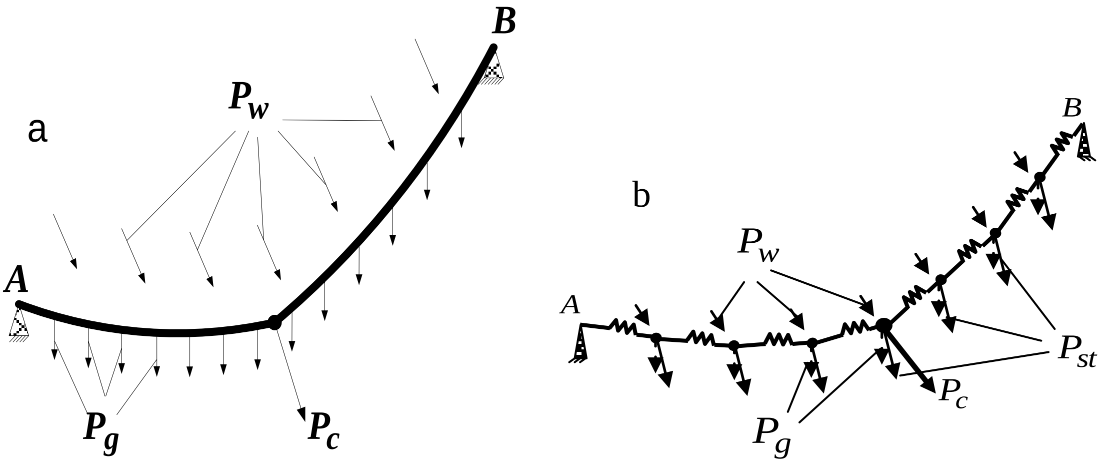

Fig. 1, a) - Rope, sagged in the gravitational field, is loaded with wind action and concentrated load; b) - its discrete model

Fig. 2 depicts the discrete model of an elementary component of rope, which, with substitution of reduced values of the parameters of mass, rigidity and damping coefficient will adequately reflect inertial, elastic and dissipative properties of rope as a whole. In other words, the parameters of elementary components are determined from the condition that the summary values of these parameters for entire model coincide with analogous values of the parameters of simulated object.

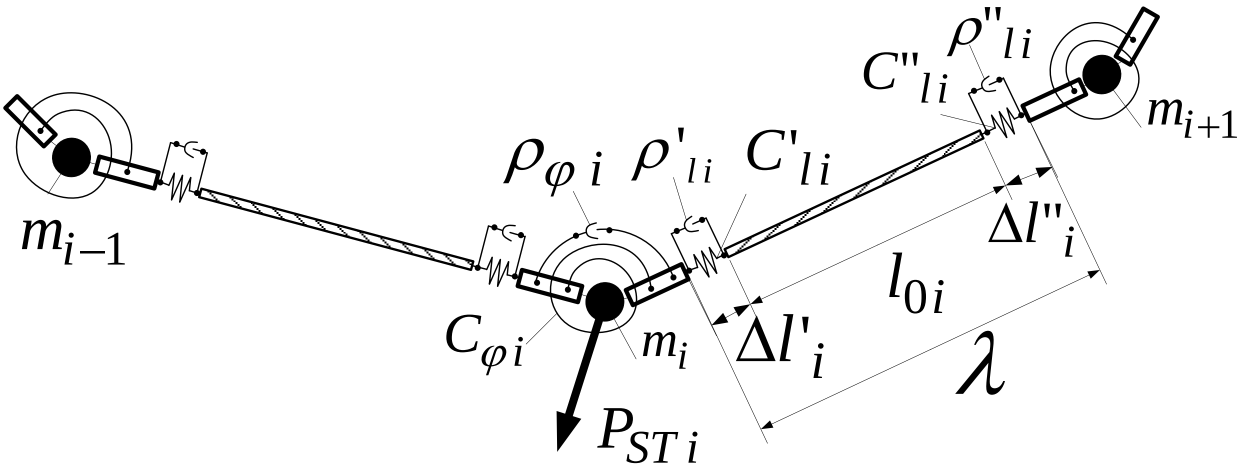

Fig. 2. Discrete model of the elementary component of rope

Mathematical description of the elementary component of rope:

$$R = \begin{cases} 0, & if \;\; \lambda \le l_0; \\ \dfrac{EF}{l_0} \cdot \Delta l, & if \;\; \lambda > l_0. \end{cases}$$

where: R is effort; E – the modulus of elasticity; F – the area of metallic section; l_0 - the length of absolutely flexible, but non inextensible section of rope (i.e. the length of the segment of rope in the unloaded state), Δl – elastic elongation, λ **–** overall length of the segment of rope**,** taking into account, in contrast to other approaches known to us, the basic property of a real, deformed rope - in the unstressed state rope has the finite length (l_0) and a small, but not in all cases negligible, bending stiffness (C_φ). The inertial and dissipative properties of rope represent point masses and dampers concentrated in the units. Assigning characteristic of springs it is possible to reflect adequately the elasticity of rope not only in longitudinal or transverse direction, but also around a longitudinal axis, i.e., torsional stiffness. Thus it is possible to simulate the property of rope – to be twisted (or untwisted) with a change in its tension.

In the model of rod (Fig. 3) the units with concentrated masses theoretically can separate infinitely from one another. The proposed model of rod does not limit approaching or, even, subsequent moving off of the units. This assumption enables to avoid possible computational problems. In unloaded state the rod has finite length l_0. In Fig. 3, b) and 3, c) is illustrated the possibility of unlimited approaching or moving off from point masses, and also the possibility of twisting of one half of the rod around the longitudinal axis. Therefore, the model in question adequately simulates the elastic properties of the rod during outstretching, compression and twisting.

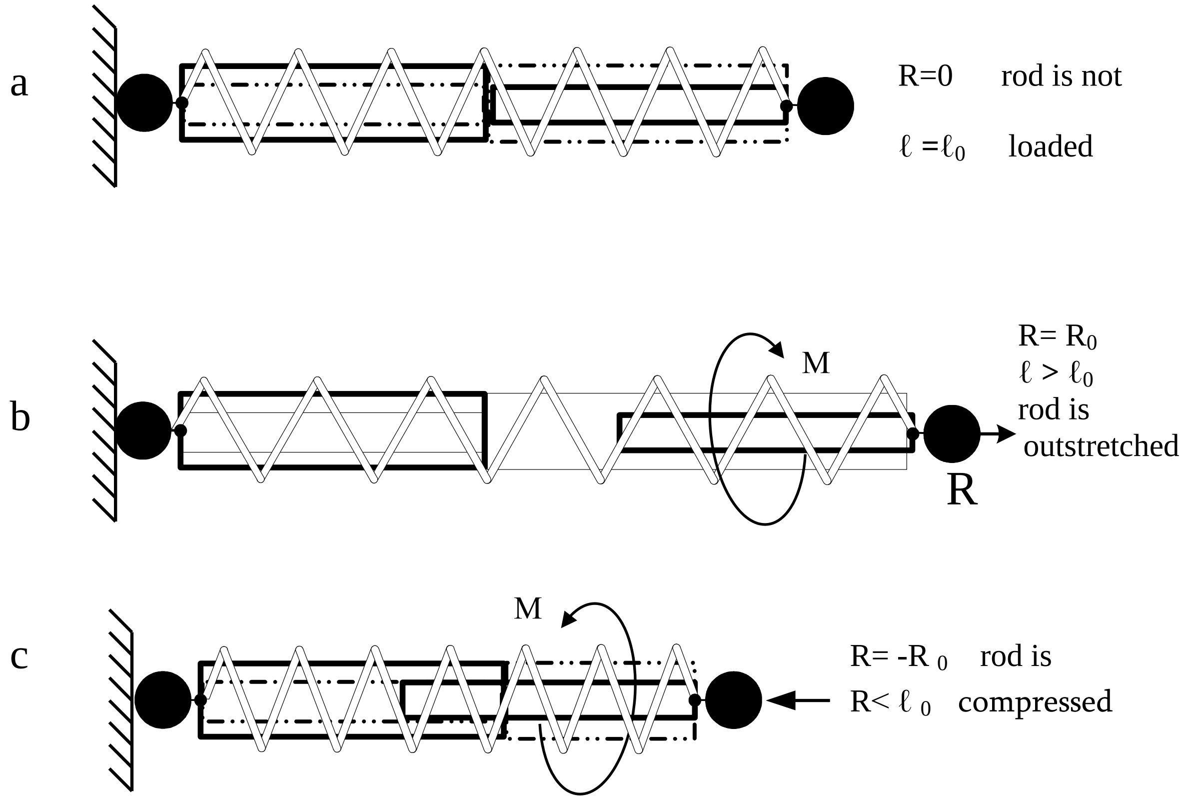

Fig. 3. Model of the rod

Analogously to the model of rope, in Fig. 4 is given a discrete model of the elementary component of rod, which contains three adjacent units of concentrated mass. Its basic (fundamental) difference from the properties of rope consists in the presence of the elastic reaction of rod to the action of compressive action and in several orders larger value of bending stiffness.

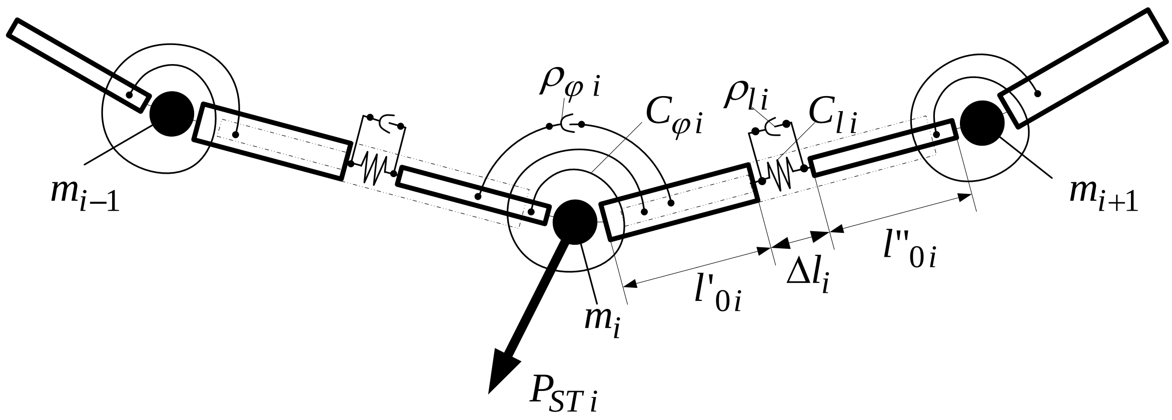

Fig. 4. Discrete model of the elementary component of the rod

For the elementary component of rod the dependence of effort on deformation will be:

$$R = \begin{cases} 0, & for \;\; \lambda = l_0; \\ \dfrac{EF}{l_0} \cdot \Delta l, & for \;\; \lambda > l_0; \\ -\dfrac{EF}{l_0} \Delta l, & for \;\; \lambda < l_0. \end{cases}$$

Here the direction of outstretch is assumed as a positive direction. If in formulas given above EF/l is replaced by the appropriate approximation of empirically obtained dependence of effort on deformation, then it is possible to simulate adequately the work of rod not only in the area of elastic deformation, but also in the area of elastic-plastic deformation.

From similar elementary components it is possible to compose a beam with any load, for example, evenly distributed throughout entire length (Pd) and with one concentrated load (Pc). This model of a beam can be also loaded with a torque (Fig. 5).

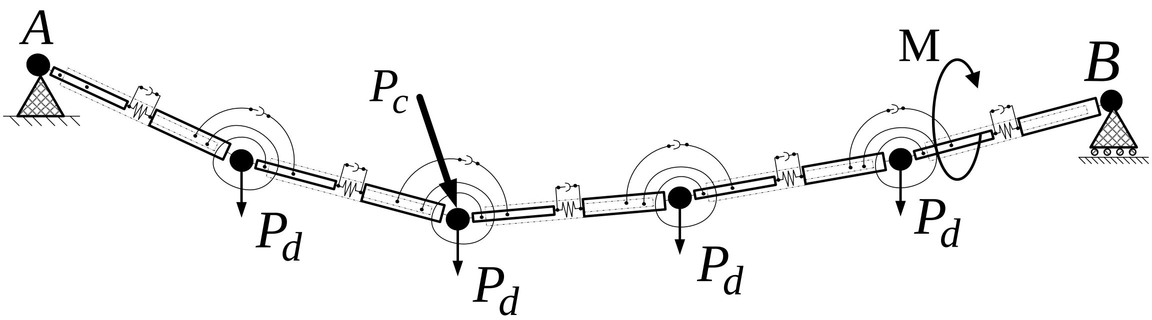

Fig. 5. Discrete model of a beam with concentrated load

The sequence of applying considering approach, by analogy, as this was done with rope and rod, implies the "decomposition" of the investigated object into standard elementary components and its representation in the form of a discrete model. Thus, the fragment of the library of standard elementary components for aerial ropeways, and also their combinations with ropes and rods, is given in Fig. 6.

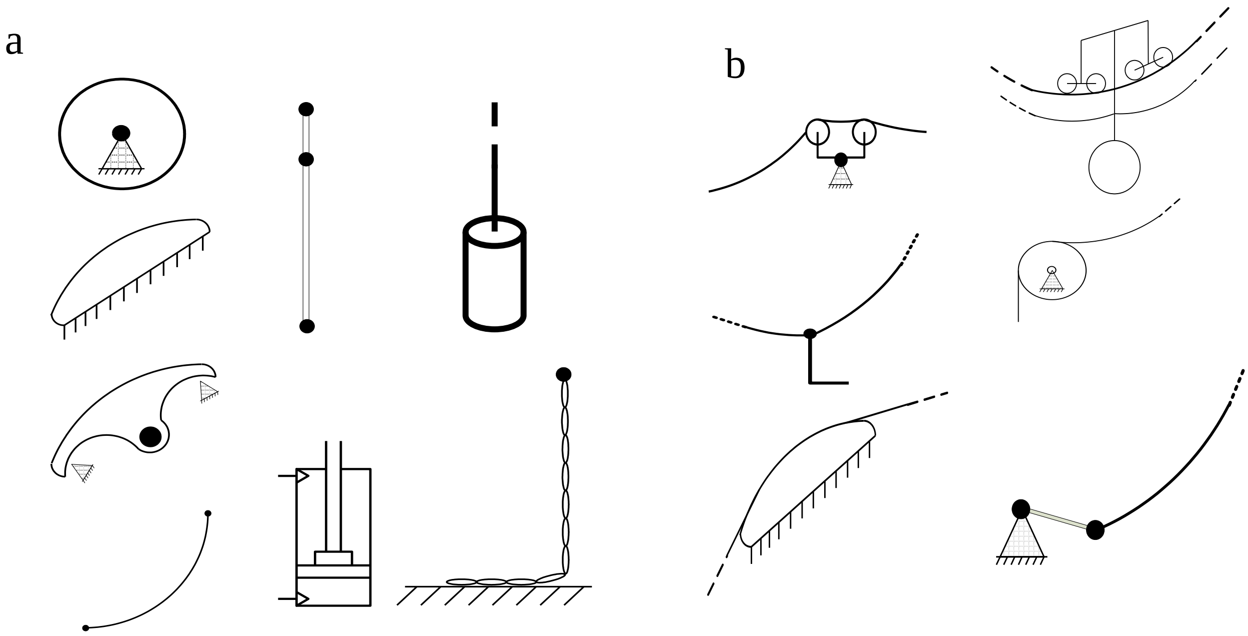

Fig. 6. Standard components of aerial ropeways (a) and their combination with ropes and rods (b)

Thus, the discrete model of suspendid pipeline passage, consisted of standard elementary components, is represented in Fig. 7.

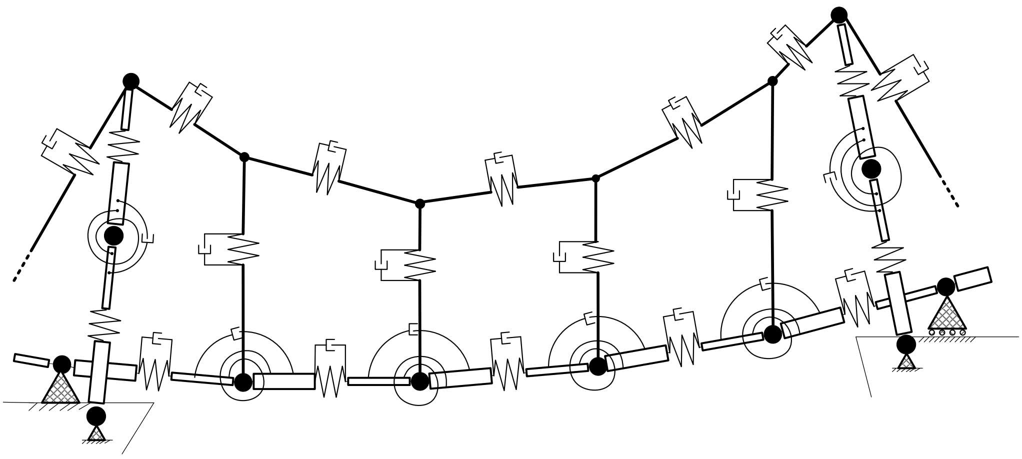

Fig. 7. Rope – rod system of suspendid pipeline passage

Analogously, with a discrete model it is possible to present adequately the inertial, elastic and dissipative properties of any construction from different spheres of engineering practice, for example, of ropeway and orbital cable system [3].

THE ALGORITHM OF CALCULATION OFF ROPE – ROAD STRUCTURE

According to the offered approach, after that it will be composed a computer image of investigating object (by word "composed" it is implied the construction of initial configuration of the object from elementary discrete components or their combinations), to the nodal points (here are, as a rule, concentrated masses) with taking into account boundary conditions will be applied all external efforts, including distributed, for example, the action of gravity.

Let us briefly describe the idea, on a basis of which the algorithm ensures determination of the state of equilibrium of the object and corresponding to it the values of parameters – efforts, displacements, geometric coordinates, etc. As the initial we assume any state, which has physical sense, and we begin with sequential iterations to process a computer model - to change the geometric configuration of rope – rod structure in such a way that it would pass into the state of equilibrium. For this purpose we divide entire system into elementary subsystems (in our case elementary components) and, taking into account boundary conditions (connections with "external world"), we process each of these subsystems independently, considering all the remaining subsystems fixed, "frozen". By processing we understand such change in the configuration, which converts the subsystem into the new state, more approximate to the state of stable equilibrium.

For this purpose, for each unit (concentrated mass) of model it is calculated the resultant force and in the direction of its vector a virtual displacement of this unit is accomplished. Thus, potential energy of the corresponding subsystem is minimized. As a result of the described repeated operations (iterations) on all subsystems without exception, configuration of the model of object will be gradually modified and, in the final analysis (in the limit), it will achieve the steady state of equilibrium (to which will correspond the minimum of potential energy of entire system). From obtained by such way geometric configuration, corresponding to the steady state of equilibrium, by inverse computations are determined the efforts, deformations and other characteristic parameters for any local place of rope – rod structure. The unconditional convergence of the process of iteration is proved in [3].

Let us note that the method of representation of investigating structure as combinations of elementary standard components, borrowed from the theory of control, enables to automate the process of composition of the model, what is especially convenient during simulation and calculation of rope – rod structure of different related constructions, for example, of aerial ropeways.

Thus, the sequence of applying the approach for simulation and calculation of rarefied rope - rod structures consists of the following basic procedures:

- the discrete representation of the object and the automatic composition ("assembling") of the model;
- the realization of the algorithm of transformation of the model for obtaining the state of equilibrium and inverse computation of efforts, displacements and other parameters in the elements of construction.

The work of considered method and the process of iteration during simulation and calculation of the separate fragments of the rope system of ropeway can be clearly traced by starting the demo-version on the site of the Institute of Mining Mechanics of Georgian Academy of Science at the address: www.mining.org.ge/develop/pataraia-dmr.

CHECKING IN PRACTICE. ADVANTAGES AND DISADVANTAGES

The checkup of constructions enumerated above [3] and a number of other classical examples from the literature on structural mechanics ([9], Fig. 8) showed the following advantages of the approach:

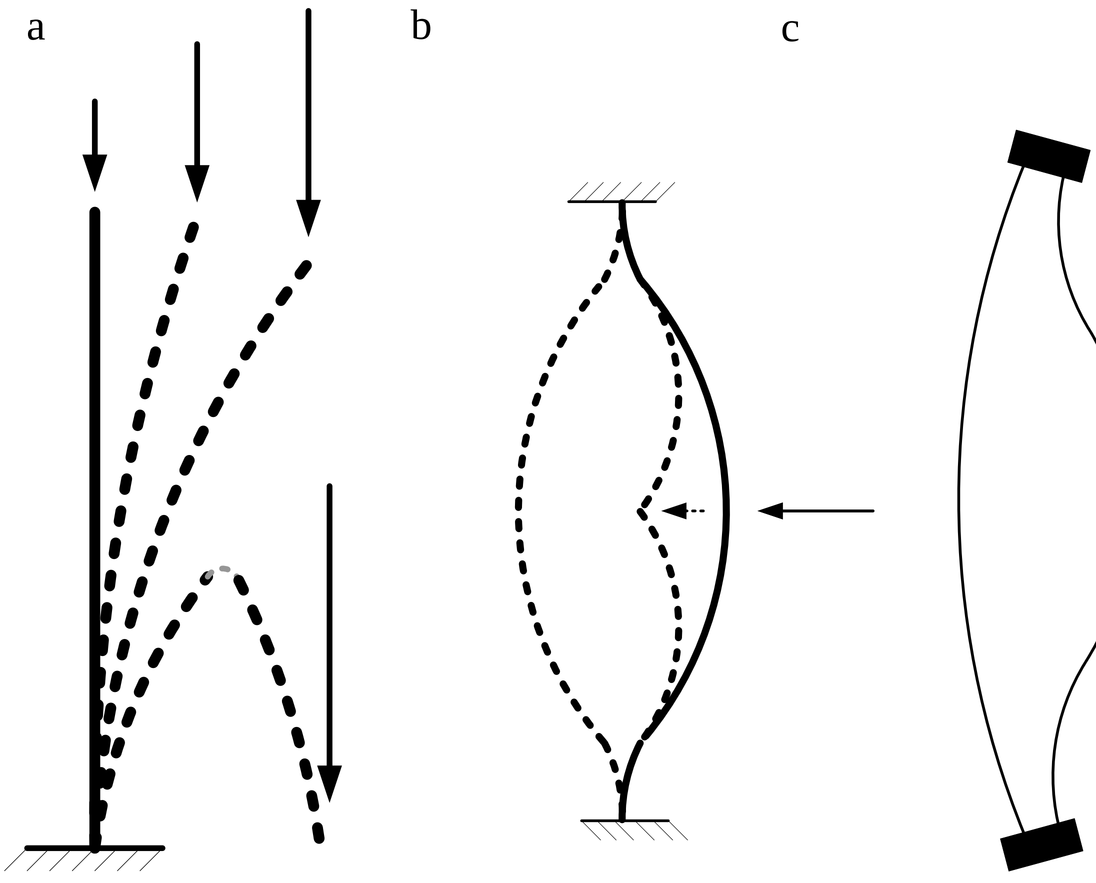

Fig. 8. Loss of stability (and "breakdown") of a rod with axial load – "Euler task" (a); passage from one steady state to another (b); the bend of two parallel rods (wires), pinched from both sides (c)

- For determining the steady state of equilibrium of construction, and, therefore, all efforts and displacements, instead of mathematical description (the system of equations) we compose physical description of object in the form of computer model and, instead of solution of the system of equations, by visual and easily realizable iterative algorithm we transform the configuration of a model to reaching the state of equilibrium.

- Universal nature – because of use of standard elementary components, almost with the same and comparatively low expenditures it is possible to simulate and to design rope – rod structure of the objects of most various configuration and complexity encountered in practice.

- It is applicable for solution of the problems of both static's and dynamics [3].

- The process of iterations converges practically in all cases and does not depend on the quality of initial approximation.

- It is easy to take into account the nonlinear characteristics of materials, friction, hysteresis, gap, discontinuity of other types, also non-stationary effects and disturbances as, for example, wind, icing, etc.

- The desired degree of accuracy can be achieved by an increase in the quantity of discrete components (point masses); by this will be increased only the expenditures of necessary machine time, but convergence of the algorithm will not suffer.

- The clarity of the representation of a model and its behavior, the ease of numerical experimentation and simulation of different emergency situations, according to which it is possible to illustrate and to observe a change in the configuration of cable system, a "breakdown" in the separate units, loss of stability (Fig. 8 a), passage from one steady form to another (Fig. 8 b), other emergency situations, for example, in the case of ropeway, descent of rope from supporting roll, break of tractive rope, etc.

For the sake of fairness it is necessary to note shortcoming of the method, which especially manifests itself during solution of multidimensional problems – long machine time, necessary for achievement of given precision. Let us note, high speed computers with multi-core processors, which appeared in wide use recently, and also new effective mathematical methods of parallel computation convert this problem into easy solvable engineering task.

Note that the most popular and widely used in the engineering practice at present the method of finite elements (MFE) [6, 7, 8] also implements the partition of investigated object into standard elements, but here, in contrast to the offered approach, the final goal – determination of displacements and efforts is reached by composition and numerical solution of the system of equations. The other significant difference, in our approach, consists in the representation of all solid deformable bodies (ropes, rods, membranes, beams, plates, etc..) as the sets of point masses, connected with rigidities (springs).

CONCLUSION

Due to application of the discrete representation and special algorithm, the described approach can be considered as sufficiently effective tool in composition and numerical investigation of complex rope-rod structures.

The advantages mentioned above and checking in practice allow considering it as a competitive in comparison with traditional approaches, although the comprehensive checking by practice will finally determine the spheres of its successful application.

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